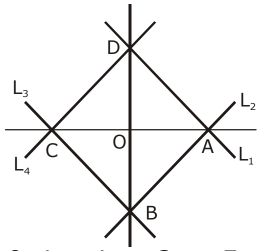
EXERCISE – III

HINTS & SOLUTIONS

Sol.1 Both pair of lines are parallel & distance between || lines are equal. So quadrilateral is square.



L_1: $3x + 4y - 5 = 0$; L_2: $4x - 3y - 5 = 0$

Intersection point of L_1 & L_2 is A,

$$\frac{x}{-20-15} = \frac{y}{-20+15} = \frac{1}{-9-16}$$

$$\Rightarrow x = \frac{-35}{-25},\ y = \frac{-5}{-25} \quad \therefore A\left(\frac{7}{5},\frac{1}{5}\right)$$

L_3 : $3x + 4y + 5 = 0$; L_4: $4x - 3y + 5 = 0$

Intersection point of L_3 & L_4 is C,

$$\frac{x}{20+15} = \frac{y}{20-15} = \frac{1}{-9-16}$$

$$\Rightarrow x = -\frac{36}{25},\ y = -\frac{5}{25} \quad \therefore C\left(\frac{-7}{5},-\frac{1}{5}\right)$$

$\because$ Centre of circle (0, 0)

$$r_{circumcircle} = \sqrt{\left(\frac{7}{5}\right)^2+\left(\frac{1}{5}\right)^2} = \sqrt{2}$$

$$r_{incricle} = \left|\frac{0\cdot x+0\cdot y-5}{\sqrt{3^2+4^2}}\right| = 1$$

Equation of circumcircle $x^2 + y^2 = 2$

& Equation of incircle $x^2 + y^2 = 1$

Sol.2 $S = 0$, centre $C_1 \equiv (-1, 1)$, radius r_1

$S_1 \equiv x^2 + y^2 - 4x + 6y - 3 = 0$

Centre $C_2 \equiv (2, -3)$, radius $r_2 = 4$

$\therefore$ S_1 & S_2 touches externally

$$\therefore C_1C_2 = r_1 + r_2 \Rightarrow \sqrt{(2+1)^2+(-3-1)^2} = r_1 + 4$$

$\Rightarrow r_1 = 5 - 4 = 1$

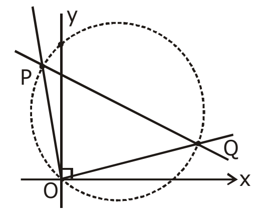
$\therefore$ Circle is $S \equiv (x + 1)^2 + (y - 1)^2 = 1^2$

$\Rightarrow S \equiv x^2 + y^2 + 2x - 2y + 1 = 0$

$\therefore$ x-intercept $= 2\sqrt{g^2-c} = 2\sqrt{1-1} = 0$

y-intercept $= 2\sqrt{f^2-c} = 2\sqrt{1-1} = 0$

Sol.3



$\ell x + my + n = 0$

$$\Rightarrow \left(\frac{\ell n+my}{-n}\right) = 1$$

By homoginization

$ax^2 + 2hxy + by^2 = (1)^2$

$$ax^2 + 2hxy + by^2 = \frac{(\ell x+my)^2}{n^2}$$

$$x^2\left(a-\frac{\ell^2}{n^2}\right) + 2xy\left(h-\frac{\ell m}{n^2}\right) + y^2\left(b-\frac{m^2}{n^2}\right) = 0$$

If represent pairs of $\perp$ lines at (0, 0) then

$\Rightarrow$ coeff. of x^2 + coeff. of y^2 = 0

$$\Rightarrow a - \frac{\ell^2}{n^2} + b - \frac{m^2}{n^2} = 0$$

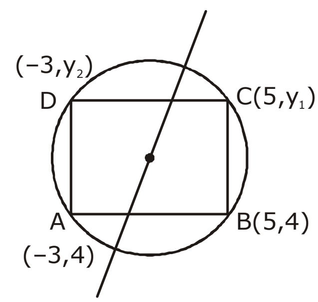
$\Rightarrow n^2 (a + b) = \ell^2 + m^2$

Sol.4 Given equation of diameter AC :

$4y = x + 7$

AB || to x-axis

mid point of AC



$$\left(-\frac{3+5}{2},\frac{4+y_1}{2}\right)$$

$$\equiv \left(1,\frac{y_1+4}{2}\right)$$

Lies on AC $\Rightarrow 4\left(\frac{y_1+4}{2}\right) = 1 + 7 \Rightarrow y_1 = 0$

$\therefore$ Centre (1, 2) so Area of ABCD = (AB)$\times$(BC)

$$= \sqrt{(5+3)^2+(4-4)^2} \times \sqrt{(5-5)^2+(4-0)^2}$$

$8 \times 4 = 32$ sq. units

Sol.5 Let circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Given $\sqrt{S_1} = 1$ $\sqrt{S_2} = \sqrt{7}$, $\sqrt{S_3} = \sqrt{2}$

$$\Rightarrow \sqrt{1^2+0^2+2g(1)+2f(0)+c} = 1$$

$\Rightarrow 2g + c + 1 = 1 \Rightarrow 2g = -c$ ....(1)

$$\& \sqrt{2^2+0^2+2g(2)+2f(0)+c} = \sqrt{7}$$

$\Rightarrow 4 + 4g + c = 7 \Rightarrow 4g + c = 3$ ....(2)

& $\sqrt{3^2+2^2+2g(3)+2f(2)+c} = \sqrt{2}$

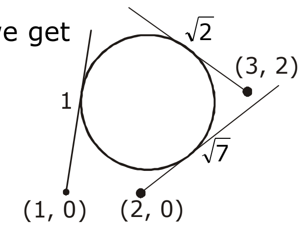
$\Rightarrow$ 6g + 4f + c + 13 = 2

$\Rightarrow$ 6g + 4f + c = −11 ...(3)

By (1) & (2) $\Rightarrow$ c = −3 & g = $\frac{3}{2}$

Put c & g in (3) we get

$\Rightarrow f = \frac{-17}{4}$



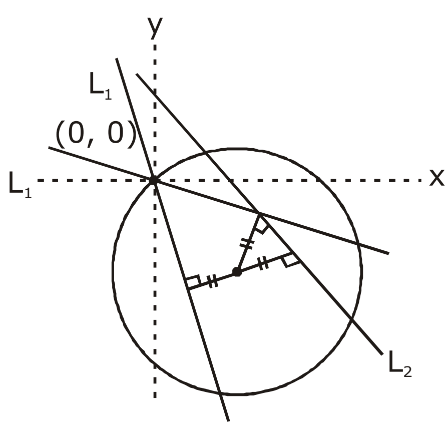
So equation of circle is

$\Rightarrow x^2 + y^2 + 3x - \frac{17}{2}y - 3 = 0$

$\Rightarrow 2(x^2 + y^2) + 6x - 17y - 6 = 0$

Sol.6 Given $x^2+y^2 - x + 3y = 0$, centre $\equiv \left(\frac{1}{2}, -\frac{3}{2}\right)$

$L_2 \equiv x + y = 1$, Let L_1 is $y = mx$



distance between centre & lines are equal

$$\frac{\left|\frac{1}{2}-\frac{3}{2}-1\right|}{\sqrt{1^2+1^2}} = \frac{\left|m\frac{1}{2}+\frac{3}{2}\right|}{\sqrt{m^2+1}} \Rightarrow \frac{2}{\sqrt{2}} = \frac{|m+3|}{2\sqrt{m^2+1}}$$

$\Rightarrow 2\sqrt{2}\sqrt{m^2+1} = |m+3|$

$\Rightarrow 8m^2 + 8 = m^2 + 6m + 9$

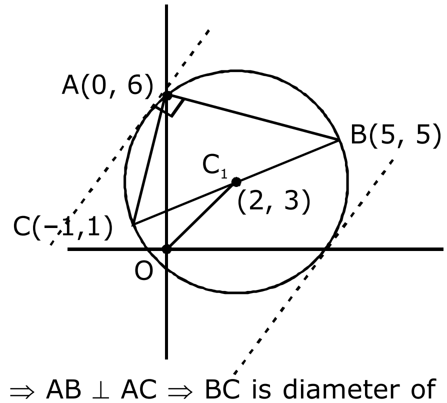
$\Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow (m-1)(7m+1) = 0 \Rightarrow m = 1$ or $\frac{-1}{7}$

$\therefore$ Equation of lines y = x or 7y + x = 0

Sol.7 Circle passes through A(0,6),B(5,5)&C(−1, 1)

$m_{AB} = -\frac{1}{5}$, $m_{AC} = 5$



$\Rightarrow AB \perp AC \Rightarrow$ BC is diameter of circle is

Centre $C_1(2, 3)$ & radius = $\sqrt{3^2+2^2} = \sqrt{13}$

Now equation of circle is

$(x-2)^2 + (y-3)^2 = (\sqrt{13})^2$

$\Rightarrow x^2 + y^2 - 4x - 6y = 0$

$m_{OC_1} = \frac{3}{2}$ Let tangent $y = \frac{3}{2}x + c$

$$\Rightarrow \sqrt{13} = \frac{\left|\frac{3}{2}.2 - 1.3 + c\right|}{\left|\sqrt{\left(\frac{3}{2}\right)^2+1}\right|} \Rightarrow \sqrt{13} \times \frac{\sqrt{13}}{2} = |c|$$

$\Rightarrow c = \pm\frac{13}{2}$ so tangents are 3x − 2y ± 13 = 0

Now tangent at (x_1, y_1) on circle

$xx_1 + yy_1 - 2(x + x_1) - 3(y + y_1) = 0$

$\Rightarrow x(x_1 - 2) + y(y_1 - 3) - 2x_1 - 3y_1 = 0$

Compair the tangents

$$\frac{x_1-2}{3} = \frac{y_1-3}{-2} = \frac{-2x_1-3y_1}{\pm 13}$$

We get points (5, 1) & (−1, 5)

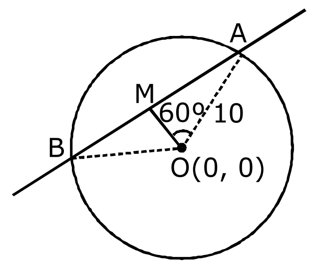
Sol.8 Combine equation of line

$(7x + y - 50) + \lambda(x - 2y - 5) = 0$

$x(7 + \lambda) + y(1 - 2\lambda) - 50 - 5\lambda = 0$

Which divides the circumference of

$x^2 + y^2 = 100$ in 2 : 1



$\Rightarrow$ angle subtend at centre will be by these segment in 2 : 1 is 240º & 120º
small angle subtend at centre is 120º
$\therefore\ \angle AOM = 60º$

$$\Rightarrow OM = \frac{|-50-5\lambda|}{\sqrt{(\lambda+7)^2+(1-2\lambda)^2}} = \frac{5|\lambda+10|}{\sqrt{5\lambda^2+10\lambda+50}}$$

$$\Rightarrow \frac{OM}{OA} = \cos 60º$$

$$\Rightarrow \frac{5|\lambda+10|}{\sqrt{5}\sqrt{\lambda^2+2\lambda+10}} \times \frac{1}{10} = \frac{1}{2}$$

$$\Rightarrow |\lambda + 10| = \sqrt{5\lambda^2+10\lambda+50}$$

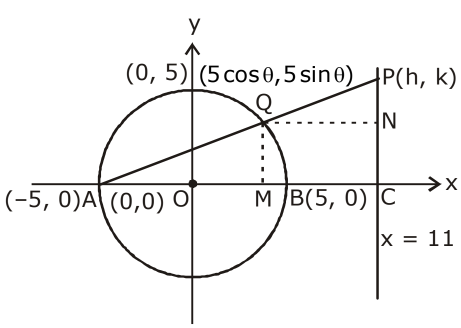
$\Rightarrow \lambda^2 + 20\lambda + 100 = 5\lambda^2 + 10\lambda + 50$
$\Rightarrow 2\lambda^2 - 5\lambda - 25 = 0$

$$\Rightarrow (\lambda - 5)(2\lambda + 5) = 0 \Rightarrow \lambda = 5, -\frac{5}{2}$$

Now equation of lines
For $\lambda=5 \Rightarrow 12x-9y-75=0$ or $4x-3y-25=0$

$$\text{For } \lambda=-\frac{5}{2} \Rightarrow \frac{9x}{2}+6y-\frac{75}{2}=0 \Rightarrow 3x+4y-25=0$$

Sol.9



(i) Area $\Delta AQB = \frac{1}{2} \times 10 \times 5 \sin\theta = 25 \sin\theta$

Area max. at $\theta = 90º \Rightarrow$ A,Q,P are collinear

$$\therefore m_{AQ} = 1 \Rightarrow \frac{k-5}{11} = 1 \Rightarrow k = 16$$

$\Rightarrow$ y = 16 so coordinate of P (11, 16)

(ii) $\frac{11-5}{2} = 5\cos\theta \Rightarrow \cos\theta = \frac{3}{5}$

$$\frac{k+0}{2} = 5\sin\theta \Rightarrow k = 10\sin\theta = 10.\ \frac{4}{5} = 8$$

$\sin\theta = \frac{4}{5}$ ($\because$ p lies above x-axis) $\therefore$ P(11, 8)

(iii) $\Delta_{AQB} = \frac{1}{4}\Delta_{APC}$

$$\Rightarrow 25\sin\theta = \frac{1}{4}\ .\ \frac{1}{2} \times 16 \times k$$

$$\Rightarrow k = \frac{25}{2}\sin\theta$$

$$m_{AQ} = m_{AP} \Rightarrow \frac{5\sin\theta}{5(\cos\theta+1)} = \frac{k}{16}$$

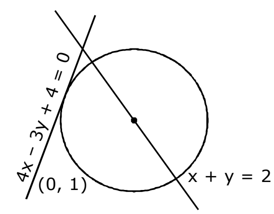
$$\Rightarrow \frac{\sin\theta}{\cos\theta+1} = \frac{25}{2.16}\sin\theta$$

$$\Rightarrow \cos\theta = \frac{32}{25} - 1$$

$$\Rightarrow \cos\theta = \frac{7}{25} \ \&\ \sin\theta = \frac{24}{25}$$

$$\therefore k = \frac{25}{2} \times \frac{24}{25} \Rightarrow k = 12 \ \therefore P(11, 12)$$

Sol.10 Let the centre of circle as the point on x+y=2 is (a, 2 –a) radius



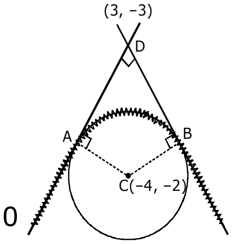
$$= \frac{|4a-6+3a+4|}{\sqrt{25}} = \frac{|7a-2|}{5}$$

$\because$ Equation of circle

$$(x - a)^2 + (y + a - 2)^2 = \frac{(7a-2)^2}{25}$$

Passing through (0, 1)
$\Rightarrow 25(2a^2 - 2a + 1) = (7a - 2)^2$
$\Rightarrow 50a^2 - 50a + 25 = 49a^2 - 28a + 4$
$\Rightarrow a^2 - 22a + 21 = 0$
$\Rightarrow (a - 21)(a - 1) = 0 \Rightarrow a = 1$ or $a = 21$
$\therefore$ centre (1,1) or (21,–19) & radius = 1 or 29
$\therefore$ Equation of circle $(x - 1)^2+(y - 1)^2 = 1^2$
$\Rightarrow x^2 + y^2 - 2x - 2y + 1$
or $(x - 21)^2 + (y + 19)^2 = 29^2$
$\Rightarrow x^2 + y^2 - 42x + 38y - 39 = 0$

Sol.11 Given $(x + 4)^2 + (y + 2)^2 = 25$
$\because$ Centre C(–4, –2)
& radius = 5
(i) Let tangent
$y + 3 = m(x - 3)$
$\Rightarrow mx - y - 3m - 3 = 0$



$$\because p = r \Rightarrow 5 = \frac{|-4m+2-3m-3|}{\sqrt{m^2+1}}$$

$\Rightarrow 5\sqrt{m^2+1} = |-7m - 1|$

$\Rightarrow 25m^2 + 25 = 49m^2 + 14m + 1$

$\Rightarrow 24m^2 + 14m - 24 = 0$

$\Rightarrow 12m^2 + 7m - 12 = 0$

$\Rightarrow (3m + 4)(4m - 3) = 0 \Rightarrow m = -\frac{4}{3}$ or $\frac{3}{4}$

So tangents are $4x+3y = 3$ & $3x - 4y = 21$

(ii) Line passes through centre $(-4, -2)$ & $\perp^{ar}$ to the tangents

$\frac{x+4}{-\frac{3}{5}} = \frac{y+2}{\frac{4}{5}} = \pm 5 \Rightarrow (-7, 2)$ or $(-1, -6)$

But $(-1, -6)$ satisfy the line $3x - 4y = 21$

& $\frac{x+5}{\frac{4}{5}} = \frac{y+2}{\frac{3}{5}} = \pm 5 \Rightarrow (0, 1)$ or $(-8, -5)$

But $(0, 1)$ satisfy the line $4x + 3y = 3$

$\therefore$ B $(-1, -6)$ & A $(0, 1)$

(iii) $\angle ADB = 90^o$

$\because$ tangent $\perp^{ar}$ to radius $CD = \sqrt{7^2+1^2} = 5\sqrt{2}$

Maximum distance of D from circle

$= r + 5\sqrt{2} = 5 + 5\sqrt{2} = 5(\sqrt{2} + 1)$units

Minimum distance of D from circle

$= |r - 5\sqrt{2}| = 5(\sqrt{2} - 1)$ units

(iv) Area of quadrilateral ADBC

= ADBC is a square

($\because$ $AD \perp BD$, $AD = BD$ & $BC = AC$)

side = 5 $\quad \therefore$ Area $= 5^2 = 25$ sq. units

ΔDAB is half of square of ADCB

$\therefore$ Area of $\Delta DAB = \frac{25}{2} = 12.5$ sq. units

(v) Cricle circumscribing the ΔDAB is a circle as diamter CD

$(x + 4)(x - 3) + (y + 2)(y + 3) = 0$

$x^2 + y^2 + x + 5y - 6 = 0$

x – intercepot

$= 2\sqrt{g^2 - c} = 2\sqrt{\frac{1}{4}+6} = 2.\frac{5}{2} = 5$

y-intercept

$= 2\sqrt{f^2 - c} = 2\sqrt{\frac{25}{4}+6} = 2.\frac{7}{2} = 7$

Sol.12 Let mid point of chord is P(h, k)

Then chord $T = S_1$ $hx + ky = h^2 + k^2$

homogenization with $x^2 - 2x - 2y = 0$

$\Rightarrow x^2 - 2(x + y)\left(\frac{hx+ky}{h^2+k^2}\right) = 0$

$\Rightarrow x^2\left(1 - \frac{2h}{h^2+k^2}\right) + y^2\left(\frac{-2k}{h^2+k^2}\right) - \frac{2(h+k)xy}{h^2+k^2} = 0$

If these line are $\perp^{ar}$ then

Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow 1 - \frac{2h}{h^2+k^2} - \frac{2k}{h^2+k^2} = 0$

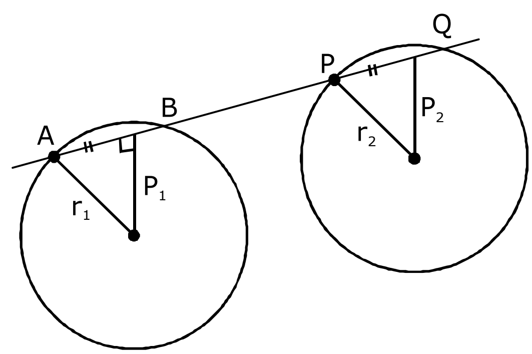
$h^2 + k^2 - 2h - 2k = 0$

$\therefore$ Locus is $x^2 + y^2 - 2x - 2y = 0$

Sol.13 Let that line is $y = x + c$

$x^2 + y^2 = 4, \Rightarrow C_1(0, 0)$ & $r_1 = 2$

$x^2 + y^2 - 10x - 14y + 65 = 0$



$\Rightarrow C_2(5, 7), r_2 = 3$

Now, $r_1^2 - p_1^2 = r_2^2 - p_2^2$

$\Rightarrow 2^2 - \frac{c^2}{2} = 3^2 - \frac{(5-7+c)^2}{2}$

$\Rightarrow \frac{8-c^2}{2} = \frac{18-(c-2)^2}{2}$

$\Rightarrow 8 - c^2 = 18 - (c - 2)^2$

$\Rightarrow 8 - c^2 = 18 - c^2 - 4 + 4c$

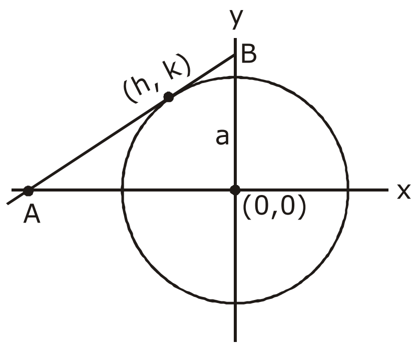
$\Rightarrow 4c = -6 \Rightarrow c = \frac{-3}{2}$

So equation of line is

$y = x - \frac{3}{2} \Rightarrow 2x - 2y = 3$

Sol.14 Tangent of $x^2 + y^2 = a^2$ is

$y = mx \pm a\sqrt{1+m^2}$



$\Rightarrow -mx + y = \pm a\sqrt{1+m^2}$

$$\Rightarrow \frac{x}{\left(\dfrac{\mp a\sqrt{1+m^2}}{m}\right)} + \frac{y}{\left(\pm a\sqrt{1+m^2}\right)} = 1$$

Let mid point of A $\left(\mp\dfrac{a\sqrt{1+m^2}}{m}, 0\right)$

& B $\left(0, \pm a\sqrt{1+m^2}\right)$ is (h, k) Then

$h = \dfrac{\mp a\sqrt{1+m^2}}{2m}$ & $k = \dfrac{\pm a\sqrt{1+m^2}}{2}$

$\Rightarrow 4m^2h^2 = a^2(1 + m^2) = 4k^2$

$\Rightarrow m^2h^2 = k^2 \Rightarrow m^2 = \dfrac{k^2}{h^2}$

$\therefore a^2\left(1 + \dfrac{k^2}{h^2}\right) = 4k^2 \Rightarrow a^2(h^2 + k^2) = 4h^2k^2$

So locus is $\Rightarrow a^2(x^2 + y^2) = 4x^2y^2$

Sol.15 Given $x^2 + y^2 = a^2$,

$x^2 + y^2 = b^2$

In ΔOPQ

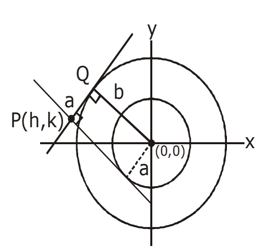
$OP^2 = OQ^2 + PQ^2$

$\Rightarrow h^2 + k^2 = b^2 + a^2$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

which is a III[rd] concentric circle

& radius $r = \sqrt{a^2 + b^2}$



Sol.16 Given L : $5x + y + 17 = 0$

& Let centre (a, b)

$(a-4)^2+(b-7)^2$...(i)

$= (a-5)^2+(b-6)^2$...(ii)

$= (a-1)^2+(b-8)^2$...(iii)

By (i) & (ii)

$\Rightarrow a - b = -2$ ...(iv)

By (ii) & (iii)

$\Rightarrow -2a + b = 1$ ...(v)

By (iv) & (v) $\Rightarrow a = 1, b = 3$

Centre C(1, 3), r = 5

So equation of circle

$(x - 1)^2 + (y + 3)^2 = 25$

$\Rightarrow x^2 + y^2 - 2x - 6y - 15 = 0$

C.O.C. from (x_1, y_1) on circle T = 0

$\Rightarrow xx_1 + yy_1 - (x + x_1) - 3(y + y_1) - 15 = 0$

$x(x_1 - 1) + y(y_1 - 3) - x_1 - 3y_1 - 15 = 0$

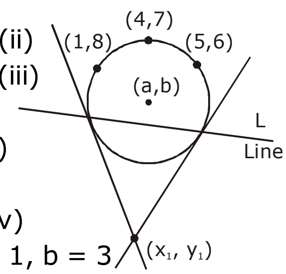
& $5x + y + 17 = 0$ are same line.

$$\therefore \frac{x_1-1}{5} = \frac{y_1-3}{1} = \frac{-x_1-3y_1-15}{17}$$

$x_1 - 5y_1 = -14$ $x_1 + 20y_1 = 36$

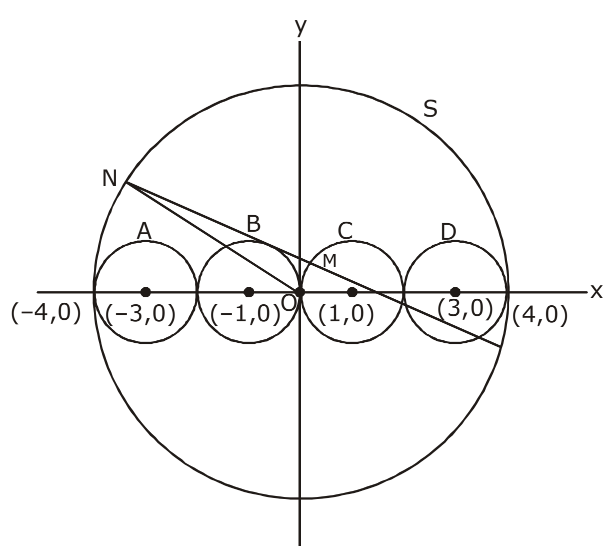
$\Rightarrow y_1 = 2, x_1 = -4$

So point of intersection of tangents(–4, 2)



Sol.17 $S \equiv x^2 + y^2 = 16$

$B \equiv (x + 1)^2 + y^2 = 1$



centre of C is (1, 0)

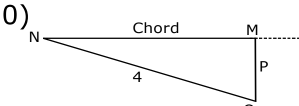
Le the chord

$y = mx + c$

passes through (1, 0) $\Rightarrow c = -m$

$\therefore y = mx - m \Rightarrow mx - y - m = 0$

chord touches B circle



$\therefore 1 = \dfrac{|-m-m|}{\sqrt{m^2+1}} \Rightarrow m^2 + 1 = 4m^2 \Rightarrow m = \pm \dfrac{1}{\sqrt{3}}$

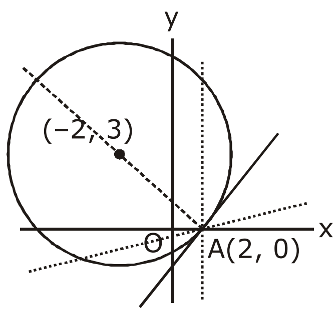
So equation of chords $x \pm \sqrt{3}\,y - 1 = 0$

$OM = \left|\dfrac{-1}{\sqrt{1+3}}\right| = \dfrac{1}{2}$

So $MN = \sqrt{ON^2 - OM^2} = \sqrt{16 - \dfrac{1}{4}} = \sqrt{\dfrac{63}{4}} = \dfrac{3\sqrt{7}}{2}$

$\therefore\ 2MN = 3\sqrt{7} = \sqrt{63} \Rightarrow x = 63$

Sol.18 A(2, 0) lie on the
given circle $(x + 2)^2 + (y - 3)^2 = 25$
$\Rightarrow x^2 + y^2 + 4x - 6y - 12 = 0$



tangent at A (2, 0) of circle is
$2 . x + 0. y + 2 (x + 2) - 3(y + 0) - 12 = 0$
$\Rightarrow 4x - 3y = 8$ ....(i)
& normal at A (2, 0) is $3x + 4y = \lambda$
passing through $(-2, 3) \Rightarrow \lambda = 6$
$\therefore 3x + 4y = 6$ ....(ii)
angle bisectors of (i) & (ii) of (tangents & normal)

$\Rightarrow \dfrac{4x - 3y - 8}{\sqrt{25}} = \pm \dfrac{(3x + 4y - 6)}{\sqrt{25}}$

$\Rightarrow x - 7y - 2 = 0$ ....(iii)

where $m = \dfrac{1}{7}$, $\cos\theta = \dfrac{7}{5\sqrt{2}}$, $\sin\theta = \dfrac{1}{5\sqrt{2}}$

& $7x + y - 14 = 0$ ....(iv)

where $m = -7$, $\cos\theta = \dfrac{-1}{5\sqrt{2}}$, $\sin\theta = \dfrac{7}{5\sqrt{2}}$

point whose distance from A is $5\sqrt{2}$ on the lines (iii) & (iv)

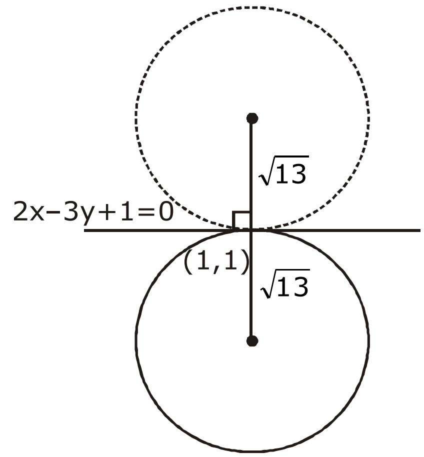
$\dfrac{x-2}{\left(\dfrac{7}{5\sqrt{2}}\right)} = \dfrac{y-0}{\left(\dfrac{1}{5\sqrt{2}}\right)} = \pm 5\sqrt{2} \Rightarrow (9, 1)\ \&\ (-5, -1)$

& $\dfrac{x-2}{\left(\dfrac{-1}{5\sqrt{2}}\right)} = \dfrac{y-0}{\left(\dfrac{7}{5\sqrt{2}}\right)} = \pm 5\sqrt{2} \Rightarrow (1, 7)\ \&\ (3, -7)$

Now equation of circles are
$(x - 9)^2 + (y - 1)^2 = 9,\ (x + 5)^2 + (y + 1)^2 = 9$
$(x - 1)^2 + (y - 7)^2 = 9,\ (x - 3)^2 + (y + 7)^2 = 9$

Sol.19 Equation of normal at (1, 1)
$3x + 2y - 5 = 0$

$\therefore \tan\theta = \dfrac{-3}{2}, \Rightarrow \cos\theta = \dfrac{-2}{\sqrt{13}}, \sin\theta = \dfrac{3}{\sqrt{13}}$



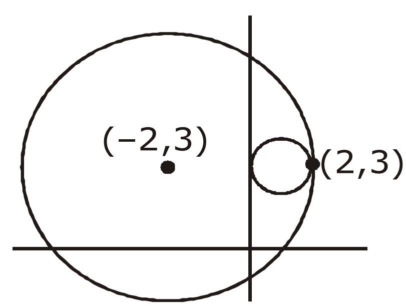
Parametric form $\dfrac{x-1}{-\dfrac{2}{\sqrt{13}}} = \dfrac{y-1}{\dfrac{3}{\sqrt{13}}} = \pm\sqrt{13}$

$\Rightarrow x = \mp 2 + 1\ \&\ y \pm 3 + 1$

$\therefore$ centres are $(-1, 4)\ \&\ (3, -2)\ \&\ r = \sqrt{13}$

So equation of circles are
$(x + 1)^2 + (y - 4)^2 = 13$
& $(x - 3)^2 + (y + 2)^2 = 13$

Sol.20 $S_1 \equiv x^2 + y^2 + 4x - 6y - 3 = 0$
& point circle $S_2 \equiv (x - 2)^2 + (y - 3)^2$
$S_1 + \lambda S_2 = 0$



$\Rightarrow x^2 + y^2 + 4x - 6y - 3 + \lambda\,[(x - 2)^2 + (y - 3)^2] = 0$
Passes through (1, 1)

$1^2 + 1^2 + 4 - 6 - 3 + \lambda(1 + 4) = 0$

$\Rightarrow -3 + 5\lambda = 0 \Rightarrow \lambda = \dfrac{3}{5}$

$\therefore 8x^2 + 8y^2 + 8x - 48y + 24 = 0$

$\Rightarrow x^2 + y^2 + x - 6y + 3 = 0$

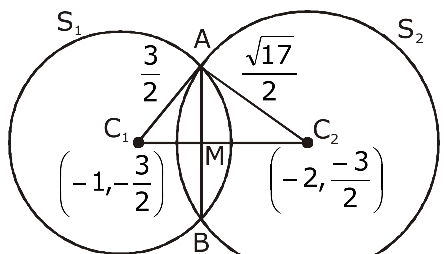
Sol.21 For K

$S_1 \equiv x^2 + y^2 + 2x + 3y + 1 = 0,$

Centre $C_1\left(-1, \dfrac{-3}{2}\right)$ & radius $r_1 = \dfrac{3}{2}$

$S_2 \equiv x^2+y^2+4x+3y+2=0,$

Centre $C_2\left(-2, \dfrac{-3}{2}\right)$ & radius $r_2 = \dfrac{\sqrt{17}}{2}$



Radical axis $S_1 - S_2 = 0$

$\Rightarrow -2x - 1 = 0 \Rightarrow 2x + 1 = 0$

$C_1M = \left|\dfrac{2(-1)+1}{\sqrt{2^2+0^2}}\right| = \dfrac{1}{2}$

$AM = \sqrt{AC_1^2 - (C_1M)^2} = \sqrt{\dfrac{9}{4} - \dfrac{1}{4}} = \sqrt{2}$

$\therefore AB = 2\sqrt{2}$ & Square of AB = 8 = k

For W

$y^2 = 8x$ & let chord is $y = mx + c$

By homoginizaztion

$\Rightarrow y^2 = 8x\left(\dfrac{y-mx}{c}\right) \Rightarrow cy^2 = 8xy - 8mx^2$

$\Rightarrow 8mx^2 - 8xy + cy^2 = 0$

If lines $\perp^{ar}$ then $8m + c = 0 \Rightarrow c = -8m$

$\therefore$ Chord is $y = mx - 8m = m(x - 8)$

So fixed point is (8, 0) $\therefore$ W = 8 + 0 = 8.

For H

Length of tangent is $L_T = \sqrt{S_1}$ & square is S_1

but Coeff. of x^2 = Coeff. of y^2 = 1

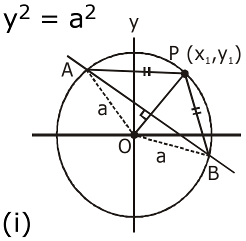
$S \equiv x^2 + y^2 + \dfrac{5}{2}y - 8 = 0$ & Point (3, 0)

$\therefore S_1 = 9 - 8 = 1$

Now, **KWH** = 8 × 8 × 1 = 64

Sol.22 PA = PB = d & $x^2 + y^2 = a^2$

$\because AB \perp OP$



$\Rightarrow m_{AB} = -\dfrac{x_1}{y_1}$

Line AB is

$xx_1 + yy_1 + c = 0$....(i)

Let A $(a\cos\theta, a\sin\theta)$

$\because AP = d \Rightarrow (x_1 - a\cos\theta)^2 + (y_1 - a\sin\theta)^2 = d^2$

$\Rightarrow x_1^2+y_1^2+a^2-2(a\cos\theta\, x_1+b\sin\theta\, y_1) = d^2$

Now, A satisfy line AB & p satisfy circle

i.e. $x_1^2 + y_1^2 = a^2$, $a\cos\theta\, x_1 + b\sin\theta\, y_1 = -c$

$\therefore 2a^2 - 2(-c) = d^2 \Rightarrow c = \dfrac{d^2}{2} - a^2$

So equation of line AB is $xx_1 + yy_1 + \dfrac{d^2}{2} - a^2 = 0$

Sol.23 $S_1 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

$S_2 \equiv x^2 + y^2 + \dfrac{3}{2}x + 4y + c = 0$ ....(ii)

Radical axis is $S_1 - S_2 = 0$

$\Rightarrow \left(2g - \dfrac{3}{2}\right)x + (2f - 4)y = 0$ is tangent of

$S_3 \equiv x^2 + y^2 + 2x - 2y + 1 = 0$ whose centre (−1,1) & radius r = 1

So $p = r \Rightarrow \dfrac{\left|\left(2g-\dfrac{3}{2}\right)(-1) + (2f-4)(1)\right|}{\sqrt{\left(2g-\dfrac{3}{2}\right)^2 + (2f-4)^2}} = 1$

$\Rightarrow -2\left(2g - \dfrac{3}{2}\right)(2f - 4) = 0 \Rightarrow g = \dfrac{3}{4}$ or $f = 2$

Sol.24 $S_1 + \lambda S_2 = 0$

$\Rightarrow (x^2 + y^2 - 4x - 6y - 12)$

$+ \lambda(x^2 + y^2 + 6x + 4y - 12) = 0$

$\Rightarrow (1 + \lambda)x^2 + (1 + \lambda)y^2 + x(6\lambda - 4)$

$+ y(4\lambda - 6) - 12\lambda - 12 = 0$

$\Rightarrow x^2 + y^2 + 2\dfrac{(3\lambda-2)x}{(\lambda+1)} + 2\dfrac{(2\lambda-3)y}{(\lambda+1)}$

$- 12\dfrac{(\lambda+1)}{(\lambda+1)} = 0$ ....(i)

& $x^2 + y^2 - 2x - 4 = 0$ ....(ii)

orthogonally (i) cut (ii)

So $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2\dfrac{(3\lambda-2)}{(\lambda+1)}\ (-1) + 2\left(\dfrac{2\lambda-3}{\lambda+1}\right)(0) = -12 - 4$

$\Rightarrow\ 3\lambda - 2 = 8\lambda + 8 \Rightarrow\ -10 = 5\lambda\ \Rightarrow \lambda = -2$

So equation of circle

$\therefore\ -x^2 - y^2 - 16x - 14y + 12 = 0$

$\Rightarrow x^2 + y^2 + 16x + 14y - 12 = 0$

Sol.25 Centre of S = 0 lie on, L : $2x - 2y + 9 = 0$

$\rightarrow$ Diaognal

cut orthogonally $x^2 + y^2 = 4$

Let any point on the line L is $\left(\lambda, \lambda + \dfrac{9}{2}\right)$ is

centre

For orthogonal condition

$2g_1(0) + 2f_1(0) = c - 4$

So equation of circle is

$x^2 + y^2 - 2\lambda x - (2\lambda + 9)\,y + 4 = 0$

$\Rightarrow\ (x^2 + y^2 - 9y + 4) + \lambda\,(-2x - 2y) = 0$

passes through the intersection of

$x^2 + y^2 - 9y + 4 = 0$ & $x + y = 0 \Rightarrow x = -y$

$\therefore\ 2y^2 - 9y + 4 = 0 \Rightarrow (y-4)(2y-1) = 0$

$\Rightarrow y = 4,\ y = \dfrac{1}{2}\ \therefore\ x = -4$ or $-\dfrac{1}{2}$

so coordinate of fixed point

$\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ & $(-4, 4)$

Sol.26 (a) Line $xy - 3x + 2y - 6 = 0$ cut circle orthogonally (i.e. line is diameter)

$\Rightarrow xy - 3x + 2y - 6 = 0$

$\Rightarrow x(y-3) + 2(y-3) = 0$

$\Rightarrow (x+2)(y-3) = 0$

Pair of lines cut orthogonally

$\therefore$ Intersection of lines is centre of circle

i.e. centre is $(-2, 3)$, passing through $(0, 0)$

$\Rightarrow r = \sqrt{13}$

So equation of circle is

$(x+2)^2 + (y-3)^2 = 13$

$\Rightarrow\ x^2 + y^2 + 4x - 6y = 0$

orthogonally cuts to $x^2 + y^2 - kx + 2ky - 8 = 0$

$2.2.\left(-\dfrac{k}{2}\right) + 2\,(-3)\,(k) = 0 - 8$

$\Rightarrow -8k = -8 \Rightarrow\ k = 1$

(b) circle cuts axes orthogonally means both axis is a diameter $\Rightarrow$ centre (0, 0)

Let cirlce $x^2 + y^2 - \lambda = 0$

& $x^2 + y^2 - 14x - 8y + 64 = 0$ are cuts orthogonally

$\therefore\ 2(0)(-7) + 2(0)(-4) = (-\lambda) + 64 \Rightarrow \lambda = 64$

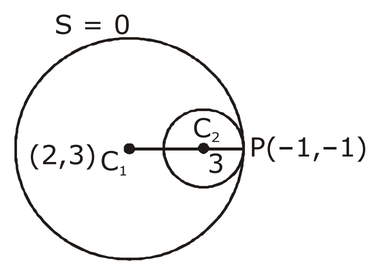
So equation of circle $x^2 + y^2 - 64 = 0$

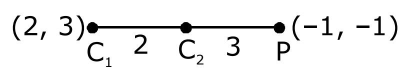
Sol.27 $S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$

centre (2, 3) & $r_1 = 5$

touches at P(–1, –1)

$r_2 = 3$





C_2 divided 2 : 3 of C_1

$\therefore\ C_2\left(\dfrac{6-2}{5}, \dfrac{9-2}{5}\right) \equiv \left(\dfrac{4}{5}, \dfrac{7}{5}\right)$

so equation of Circle is

$\left(x - \dfrac{4}{5}\right)^2 + \left(y - \dfrac{7}{5}\right)^2 = 3^2$

$\Rightarrow\ 25(x^2 + y^2) - 40x - 70y + 16 + 49 = 225$

$\Rightarrow\ 5x^2 + 5y^2 - 8x - 14y - 32 = 0$

Sol.28 Let a circle S = 0 cuts two circles $S_1 = 0$ & $S_2 = 0$ orthogonally

$2gg_1 + 2ff_1 = c + c_1$

$2gg_2 + 2ff_2 = c + c_2$

$2g(g_1 - g_2) + 2f(f_1 - f_2) = c_1 - c_2$

locus of centre $(-g, -f)$ is

$-2x(g_1 - g_2) - 2y\,(f_1 - f_2) + (c_1 - c_2) = 0$

which is straight line & radical axis of S_1 & S_2

locus of centre of circle is radical axis

$x^2 + y^2 + 4x - 6y + 9 = 0$

$x^2 + y^2 - 5x + 4y + 2 = 0$

$-\quad -\quad +\quad -\quad -$

$9x - 10y + 7 = 0$ radical axis